position

Radius 1   Radius 2

S   E

r

M

$(N-1)^2 r$

Radius1: 60
Radius2: -45
Position: -135
Velocity: 5
Resolution: 90

Hide
Clear
Reset
Draw

Red: 0
Green: 0
Blue: 0

x(t)=15cos(t)+135cos(15t/45)   x(t)=15sin(t)-135sin(15t/45)
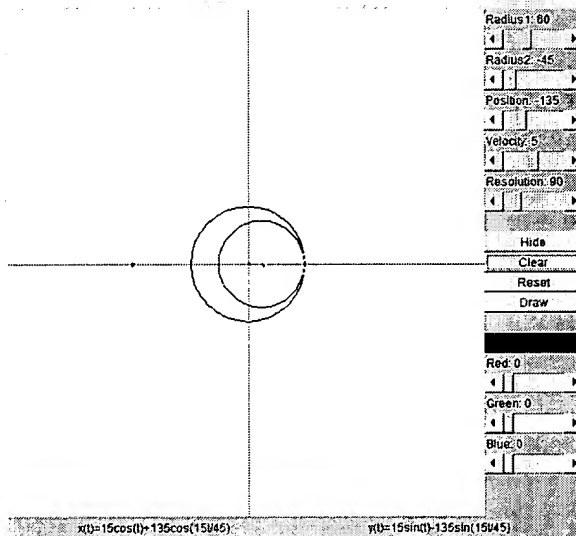
Claims 1 and 6.

SpiroGraph

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- **Velocity**
  Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!
- **Resolution**
  Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 360, the points $(x(t), y(t))$ are plotted for t=0, 1, 2, 3, 4, 5, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 8, 10, ..., 358, 360 are used. Be careful, the larger the Resolution, the longer it will take to draw the SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Radius2 (say Radius2=1), a small change in Resolution can result in remarkably different SpiroGraphs.
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  This button stops the current SpiroGraph and returns Circle2 and the Point to its original position
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Radius1: 60
◀ ▯ ▶

Radius2: -45
◀ ▯ ▶

Position: -135
◀ ▯ ▶

Velocity: 5
◀ ▯ ▶

Resolution: 90
◀ ▯ ▶

| Hide |
| Clear |
| Reset |
| Draw |

Red: 0
◀ ▮ ▶

Green: 0
◀ ▮ ▶

Blue: 0
◀ ▮ ▶

x(t)=15cos(t)+135cos(15t/45)   y(t)=15sin(t)-135sin(15t/45)

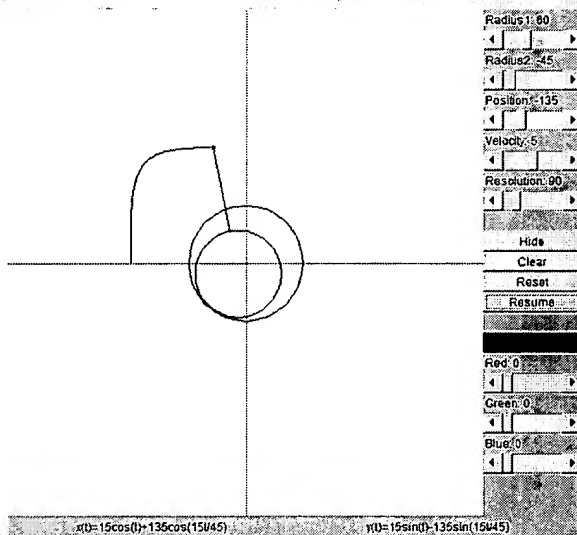| SIMPLE CURVES	Radius1	Radius2	Position	Resolution	Try this:
Cardioid	60	60	60	270	See what happens when you vary the position to the left or to the right. How could you get a cardioid oriented differently?
Astroid	60	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.
Four-leaved rose	60	-15	45	270	Try to vary the radius and position to get 3,5 or 6 leaves
Vertical Line	60	-30	-30	270	What would you change to get a horizontal line?
Ellipse	60	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	-101	270	How about a rounded triangle or pentagon?
Goldfish	75	-25	85	270	This is supposed to be fun, right?

SpiroGraph

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Radius1: 60
Radius2: 45
Position: -135
Velocity: 5
Resolution: 90
Hide
Clear
Reset
Resume
Red: 0
Green: 0
Blue: 0

x(t)=15cos(t)+135cos(15t/45)   y(t)=15sin(t)-135sin(15t/45)

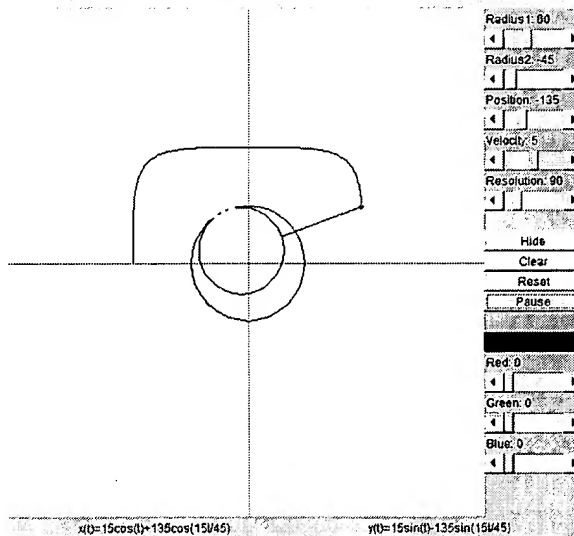| SIMPLE CURVES	Radius1	Radius2	Position	Resolution	Try this:
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Four-leaved rose	60	-15	45	270	Try to vary the radius and position to get 3,5 or 6 leaves
Vertical Line	60	-30	-30	270	What would you change to get a horizontal line?
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Radius1: 60

Radius2: -45

Position: -135

Velocity: 5

Resolution: 90

Hide

Clear

Reset

Pause

Red: 0

Green: 0

Blue: 0

x(t)=15cos(t)+135cos(15t/45)   y(t)=15sin(t)-135sin(15t/45)

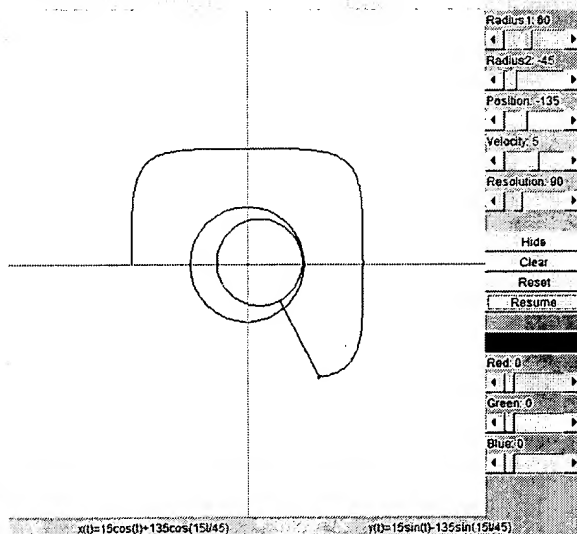| SIMPLE CURVES	Radius1	Radius2	Position	Resolution	Try this:
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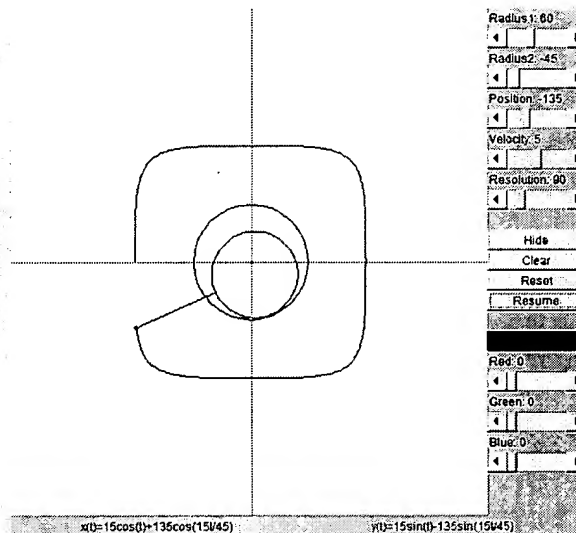| SIMPLE CURVES	Radius1	Radius2	Position	Resolution	Try this:
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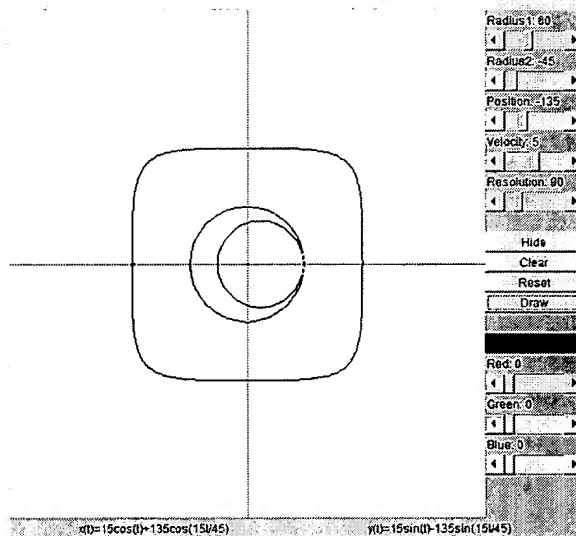| SIMPLE CURVES	Radius1	Radius2	Position	Resolution	Try this:
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